Subject	class	Paper	Study materials on	Resource Person
Mathematics	D-I (H)	I	Transformation of Cubic eq^n in the form $z^3+3Hz+G=0$	Dr. S. Ahmad

Q. Reduce the cubic equation $a_0x^3+3a_1x^2+3a_2x+a_3=0$ in the form $z^3+3Hz+G=0$ where H and G have their usual meaning.

Solution The given cubic equation is

$$f(x) \equiv a_0x^3+3a_1x^2+3a_2x+a_3=0 \quad \text{——— (1)}$$

We diminish the roots of $f(x)=0$ by a constant h and resulting equation is obtained by putting $y=x-h$

$\Rightarrow \quad x = y+h$ in (1), Therefore

$$a_0(y+h)^3+3a_1(y+h)^2+3a_2(y+h)+a_3=0$$

$$\Rightarrow a_0(y^3+3y^2h+3yh^2+h^3)+3a_1(y^2+2yh+h^2)$$
$$+3a_2(y+h)+a_3=0$$

$$\Rightarrow a_0y^3+3(a_0h+a_1)y^2+3(a_0h^2+2a_1h+a_2)y$$
$$+(a_0h^3+3a_1h^2+3a_2h+a_3)=0$$

$$\Rightarrow A_0y^3+3A_1y^2+3A_2y+A_3=0 \quad \text{——— (2)}$$

where $A_0=a_0$

$A_1=(a_0h+a_1)$

$A_2=(a_0h^2+2a_1h+a_2)$

$A_3=a_0h^3+3a_1h^2+3a_2h+a_3$

If in the transformed equation, the second term may be absent, then $A_1=0 \Rightarrow a_0h+a_1=0 \Rightarrow a_0h=-a_1$

$$\therefore h=\left(\frac{-a_1}{a_0}\right)$$

$$\therefore A_2 = a_0 h^2 + 2a_1 h + a_2$$

$$= a_0\left(\frac{-a_1}{a_0}\right)^2 + 2a_1\left(\frac{-a_1}{a_0}\right) + a_2$$

$$= \frac{\cancel{a_0} a_1^2}{a_0^{\cancel{2}}} - \frac{2a_1^2}{a_0} + a_2$$

$$= \frac{a_0 a_2 - a_1^2}{a_0} = \frac{H}{a_0}$$

$$A_3 = a_0 h^3 + 3a_1 h^2 + 3a_2 h + a_3$$

$$= a_0\left(\frac{-a_1}{a_0}\right)^3 + 3a_1\left(\frac{-a_1}{a_0}\right)^2 + 3a_2\left(\frac{-a_1}{a_0}\right) + a_3$$

$$= -\frac{\cancel{a_0} a_1^3}{a_0^{\cancel{3}2}} + \frac{3a_1^3}{a_0^2} - \frac{3a_1 a_2}{a_0} + a_3$$

$$= \frac{a_0^2 a_3 + 3a_1^3 - a_1^3 - 3a_0 a_1 a_2}{a_0^2}$$

$$= \frac{a_0^2 a_3 + 2a_1^3 - 3a_0 a_1 a_2}{a_0^2} = \frac{G}{a_0^2}$$

& $A_0 = a_0$

Hence the transformed equation (2) becomes

$$a_0 y^3 + \frac{3H}{a_0} y + \frac{G}{a_0^2} = 0$$

$$\Rightarrow y^3 + \frac{3H}{a_0^2} y + \frac{G}{a_0^3} = 0$$

If the roots of this equation be multiplied by a_0 and the transformed equation is $z^3 + 0.a_0 z^2 + a_0^2 \cdot \frac{3H}{a_0^2} \cdot z + a_0^3 \frac{G}{a_0^3} = 0$

$$\Rightarrow \boxed{z^3 + 3Hz + G = 0}$$

It is required form.